ANALYSIS OF NOISE IN ROOM-TEMPERATURE MILLIMETER-WAVE MIXERS

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Abstract

Noise in room-temperature millimeter-wave mixers consists almost entirely of shot and thermal noise generated by the mixer diode. The shot noise is shown to have down-converted components which are correlated with one another. This correlation explains the previously observed "anomalous" noise in millimeter-wave mixers.

Introduction

In order to predict accurately the performance of a Schottky diode mixer it is necessary to possess: (i) a thorough knowledge of the embedding impedance (Z_e) as seen by the diode, (ii) an accurate representation of the diode equivalent circuit, (iii) a means of performing a nonlinear analysis to determine the waveforms existing within the mixer, and (iv) appropriate models for the small-signal conversion and noise processes within the mixer. Of the several requirements, the noise model is the most challenging, since until now, an accurate analysis of millimeter-wave mixer noise has not been presented.

Previous noise models have predicted far less noise than is typically observed, and have led several investigators to the concept of an "anomalous" noise source. The purpose of this paper is to show that there is no anomalous noise present in room-temperature mixers and that mixer noise originates almost entirely from shot and thermal sources, with a small additional contribution due to phonon scattering [1].

The Generalized Mixer Noise Equations

In order to calculate the small-signal properties of a mixer the pumped diode is characterized as a linear network which can be described by its Y- or Z-matrix [2], with each port of the network representing one of the small-signal frequencies (see Figure 1). Thus:

$$\underline{\delta I} = \underline{\underline{Y}}\ \underline{\delta V} \quad , \tag{1}$$

where[1]

$$\underline{\delta I} = [\ldots\delta I_1,\ \delta I_0,\ \delta I_{-1}\ldots]^T \quad ,$$

and

$$\underline{\delta V} = [\ldots\delta V_1,\ \delta V_0,\ \delta V_{-1}\ldots] \quad .$$

The elements of the Y-matrix are given by,

$$Y_{mn} = G_{m-n} + j(\omega_0 + m\omega_p)\ C_{m-n} \quad , \tag{2}$$

where G_k and C_k are the Fourier coefficients of the large-signal diode conductance and capacitance waveforms. It is convenient to form an augmented Y-matrix $\underline{\underline{Y}}'$, which is the admittance matrix of the multiport network (indicated by the broken line in Figure 1) representing the complete mixer, including all its external terminating impedances Z_{e_m}. The augmented Y-matrix is related to the conventional Y-matrix via:

$$\underline{\delta I}' = \underline{\underline{Y}}'\ \underline{\delta V} \quad , \tag{3}$$

where the elements of $\underline{\delta I}'$ are defined in Figure 1, and

$$\underline{\underline{Y}}' = \underline{\underline{Y}} + \text{diag}\left[\frac{1}{Z_{e_m} + R_s}\right] \quad . \tag{4}$$

Inverting Equation 3 yields:

$$\underline{\delta V} = \underline{\underline{Z}}'\ \underline{\delta I}' \quad . \tag{5}$$

where $Z' \triangleq (Y')^{-1}$.

If the Z'-matrix of a particular mixer is known, the output noise voltage, δV_{out}, can be calculated from Equation 5 provided that the equivalent input current noise sources, $\underline{\delta I}'$, are also known. Measurements can be made of the equivalent output noise-temperature, T_{out},[2] where T_{out} is related to δV_{out} by

$$k\ T_{out}\ \Delta f \triangleq \frac{\langle \delta V_{out}{}^2 \rangle}{4\text{Re}\{Z_{out}\}} \quad , \tag{6}$$

where

$$\langle \delta V_{out} \rangle^2 = \langle \delta V_{out}\ \delta V^*_{out} \rangle = \underline{Z}'\ \underline{\underline{\langle \delta I'\ \delta I'^{\dagger} \rangle}}\ \underline{Z}'^{\dagger} \quad , \tag{7}$$

Z_{out} is the mixer output impedance, $\underline{Z}'$ is the individual row of the Z'-matrix which describes the conversion from the various small signal sideband frequencies to the intermediate frequency, and $\underline{\underline{\langle \delta I'\ \delta I'^{\dagger} \rangle}}$ is a *correlation matrix* describing the noise process within the mixer. Assuming that the Z'-matrix is known it remains only to evaluate the correlation matrix in order to calculate the mixer output temperature, since the output impedance can be derived directly from the Z'-matrix.

Evaluation of the Correlation Matrix

We shall initially consider only thermal and shot noise. Thermal noise is generated principally within the parasitic series resistance associated with the ideal diode, and is statistically independent of the shot noise and the conversion process occurring within the mixer. Therefore, thermal noise generated at each sideband frequency is independently down-converted by the action of the mixer and combines in quadrature at the intermediate frequency. This can be expressed mathematically by:

$$\langle I'_t\ I'^{\dagger}_t \rangle_{mn} = \frac{4kT\ R_s\ \Delta f}{|Z_{e_m} + R_s|^2} \qquad m=n \quad ,$$
$$= 0 \qquad m \neq n \quad . \tag{8}$$

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[1] The subscript notation for small-signal sideband frequencies follows that of Saleh [9]: subscript m indicates frequency $\omega_0 + m\omega_p$, where ω_0 and ω_p are the intermediate and LO frequencies.

[2] The equivalent input noise temperature, T_M, is related to T_{out} via the conversion loss (L), $T_M = LT_{out}$; where L is a function of elements of the Z'-matrix, the signal source impedance, and the series resistance [1].

Thus the thermal noise correlation matrix is diagonal, reflecting the absence of cross-correlation between the down-converted thermal-noise components originating at the various sideband frequencies.

Shot noise, on the other hand, has down-converted components which are correlated with one another. Since the probability of an electron crossing the diode's potential barrier (and thereby generating a "shot" of noise) is proportional to the instantaneous forward current through the ideal diode element, the shot noise is partially correlated with the local oscillator current waveform of the diode (and thus with the diode conversion mechanism). The shot noise correlation matrix is not therefore diagonal, and has been shown by Dragonne [3] to have elements

$$\langle I'_s I'^{\dagger}_s \rangle_{mn} = 2q\, I_{m-n}\, \Delta f \quad , \qquad (9)$$

where I_k is the k-th Fourier coefficient of the large-signal current flowing in the diode conductance.

The total correlation matrix, including both shot and thermal noise as required for the evaluation of Equation 7, is simply the sum of equations 8 and 9, reflecting the independence of the shot noise and thermal noise processes. Equations 6-9 constitute a noise model which accounts for shot and thermal noise in an arbitrarily terminated millimeter-wave mixer, in which mixing is accomplished by the simultaneous action of the diode's conductance and capacitance. This model differs significantly from its predecessors in that it explicitly considers the correlated nature of the shot noise, and although this concept is not new [3-6], this is the first time that a general noise analysis has been combined with a general conversion loss analysis resulting in a procedure which accurately predicts millimeter-wave mixer performance.

Experimental Verification

The above analysis has been successfully verified by predicting the noise performance of a millimeter-wave mixer designed to operate over the frequency range of 80-120 GHz. The mixer uses a GaAs Schottky barrier diode and is described in Ref. 7. Before an accurate analysis could be performed it was necessary to know the embedding impedance seen by the diode at a number of significant frequencies, as well as the equivalent circuit of the diode valid in the vicinity of 100 GHz (see Fig. 2). The techniques used to determine this information are explained in Ref. 1. A non-linear analysis is used to determine the Fourier coefficients, G_k and C_k, of the diode's large-signal conductance and capacitance waveforms, and these coefficients are used to determine the Y-matrix (Equation 2), and the shot noise correlation matrix (Equation 9).

Theoretical results derived via the above analysis are compared in Figure 3 with experimentally measured results at 87 GHz. The tolerances shown for the theoretical analysis reflect the uncertainties in the diode equivalent circuit parameters at the frequencies of interest; however it is clear that the present theory is substantially verified by the experiment. Also shown in Figure 3 are the results of a previous analysis [7] which led one of us to postulate an anomalous noise source [8] to account for the discrepancy between the earlier theory and experiment.

Sources of Mixer Loss and Noise

The successful analysis of millimeter-wave mixer performance makes possible the identification of the sources of mixer loss and noise. Figure 4 illustrates the breakdown of the loss, L, and the equivalent input noise temperature, T_M, as functions of the back-short position (and thus the embedding impedance) in an 87 GHz mixer. Noise in this room-temperature mixer is composed almost entirely of shot and thermal noise, with a small additional contribution due to phonon scattering in the series resistance. Scattering noise is generated during the part of the LO cycle in which a large instantaneous current flows through the diode [1]. It can be seen that in the vicinity of the minimum of T_M, the contributions due to shot and thermal effects are almost equal, and the contribution due to the scattering process is fairly small.

Conclusion

Noise in room-temperature millimeter-wave mixers is shown to originate mainly as shot and thermal noise in the diode, with a secondary contribution from scattering effects occurring in the semiconductor material. The present analysis shows that the "anomalous" noise hitherto observed in room-temperature mixers, is due primarily to the correlation properties of down-converted shot noise. In cryogenically cooled mixers, however, scattering noise will probably have a significant effect on performance and may account for the excessive noise observed in these devices [10].

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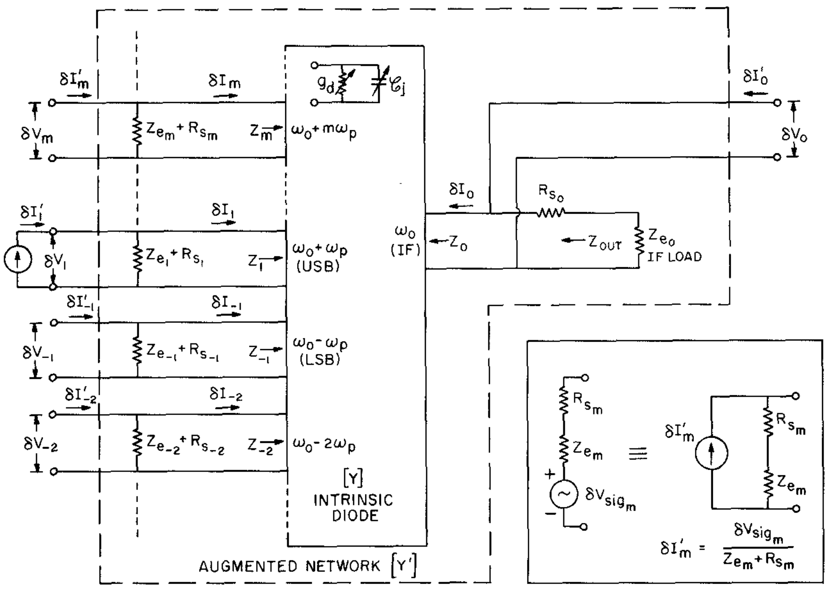
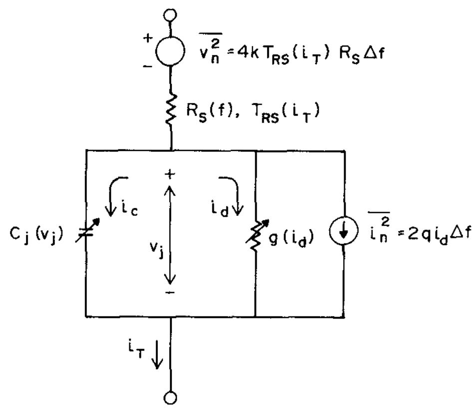


Fig. 1. Small-signal representation of the mixer as a multi-frequency linear multi-port network. The voltage and current δV_m and δI_m, at any port m are the small-signal components at frequency $(\omega_0 + m\omega_p)$ appearing at the intrinsic diode; each port represents one sideband frequency. The conversion matrix $\underline{\underline{Y}}$ is the admittance matrix of the intrinsic diode. The augmented network (broken line) includes all the sideband embedding impedances Z_{e_m}, and is characterized by the augmented admittance matrix $\underline{\underline{Y}}'$. During normal mixer operation the equivalent signal current generator $\delta I'$ is connected at port 1 of the augmented network, the other ports being open-circuited. In the noise analysis, equivalent noise current sources, $\delta I'_{s_m}$ and $\delta I'_{t_m}$, are connected to all ports. The inset shows the relation between the signal source δV_{sig_m} at the m-th sideband, and its equivalent current source $\delta I'_m$.



TYPICAL VALVES
FOR 2.5μ DIODES

i_0	8.3×10^{-17}A
η	1.05
Cjo	7×10^{-15} F
ϕ	0.95
$R_{S\,DC}$	9.4Ω
$R_{S_{115GHz}}$	12$-$16Ω
$\gamma(V_j)$	0.3$\rightarrow$0.5
T_{RS}	>298°K, A FUNCTION OF CURRENT

Fig. 2. Equivalent circuit of millimeter-wave Schottky barrier diode. The circuit shown, described in [1], is somewhat different from the conventional representation. In particular the series resistance is allowed to increase with frequency due to the skin effect, and the temperature of the resistor is allowed to vary with the instantaneous current through it due to phonon scattering effects.

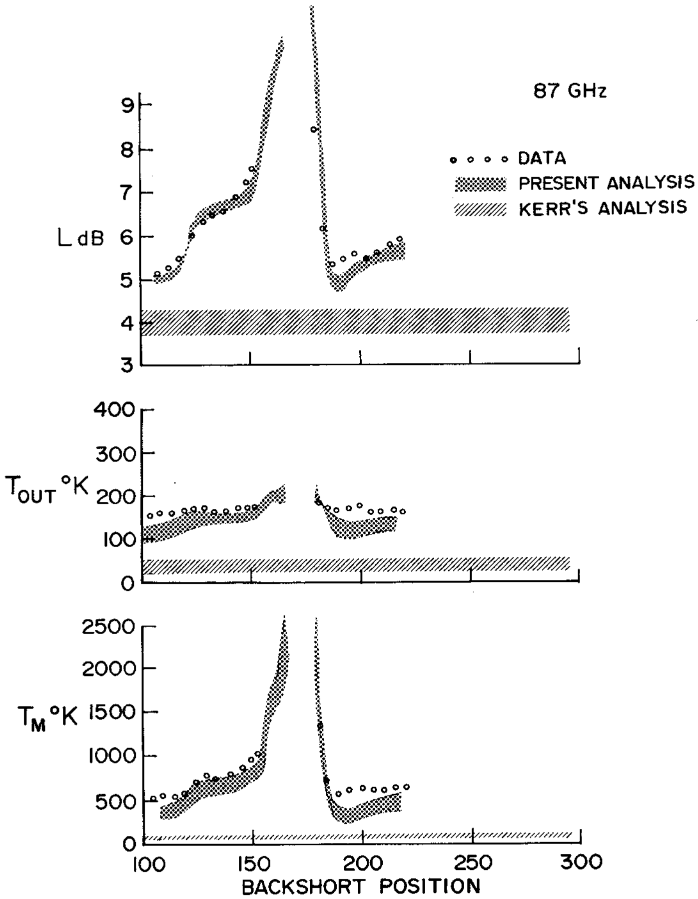


Fig. 3. Comparison of present theory and experiment. Also shown is Kerr's [8] earlier analysis which significantly underestimates the noise generated in millimeter-wave mixers.

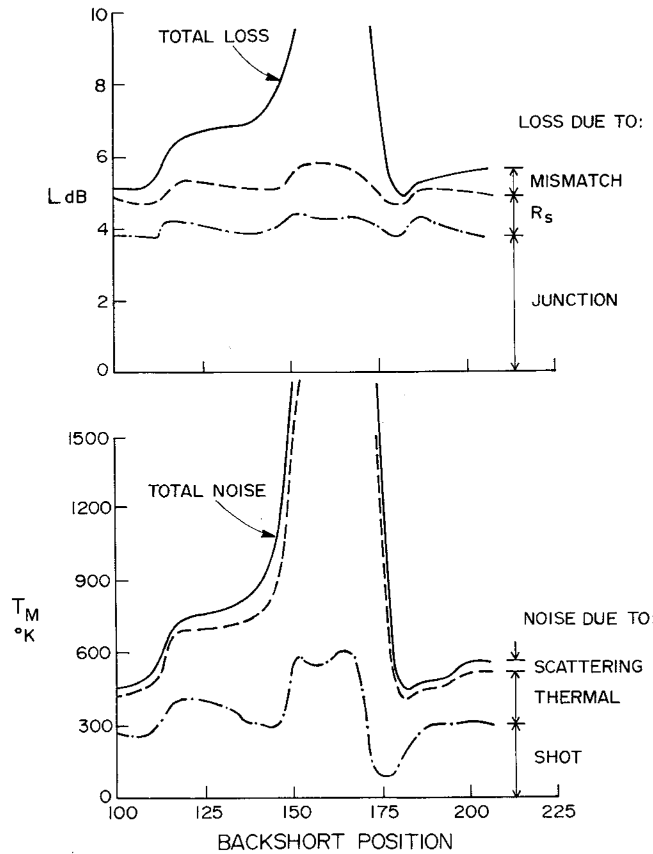


Fig. 4. Breakdown of loss and noise in a millimeter-wave mixer. The noise is essentially composed of shot and thermal components, with a slight additional contribution due to scattering in the diode series resistance.